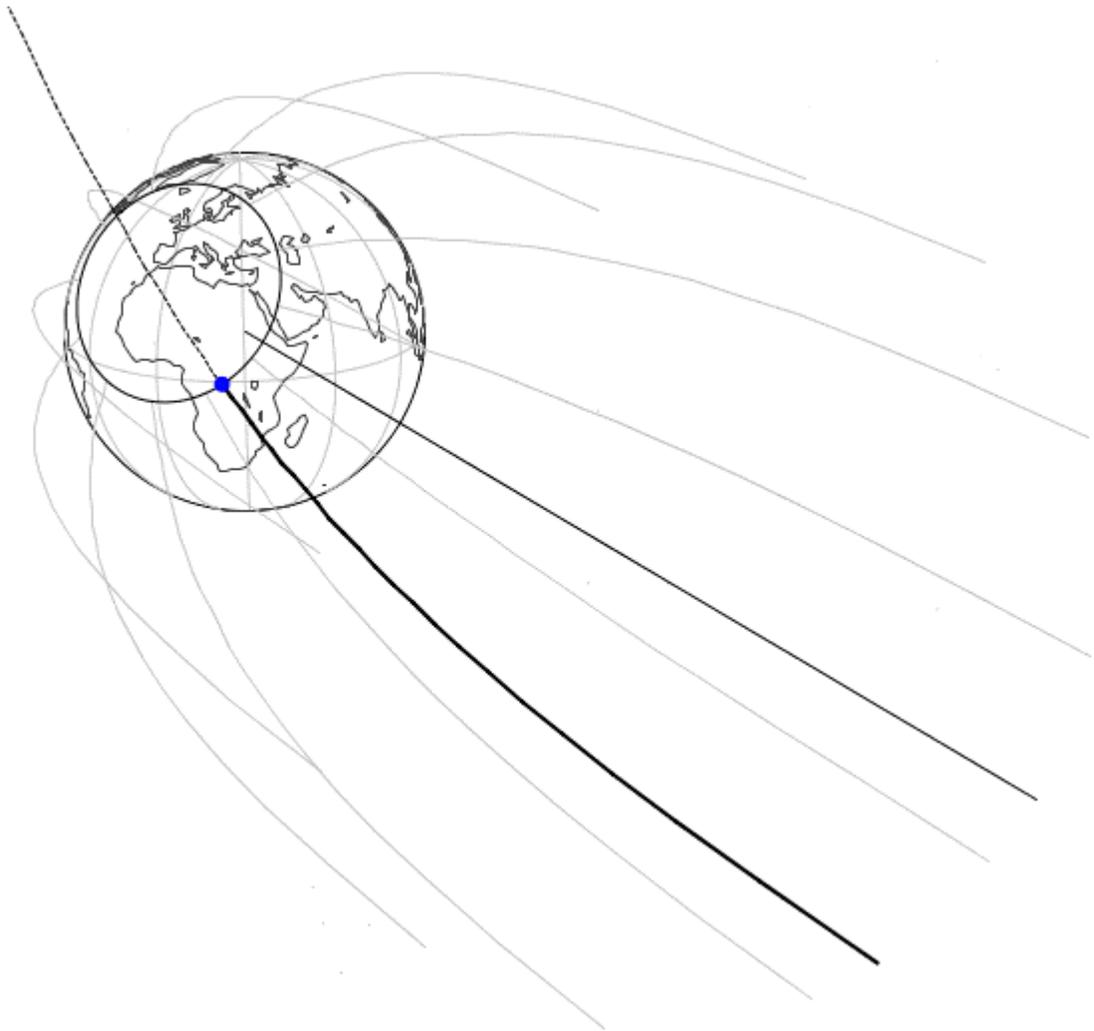# AIAA 2003-5221
Hyperbolic Injection Issues for MXER Tethers

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Hyperbolic Injection Issues for MXER-assisted payloads

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Momentum-exchange/electrodynamic reboost (MXER) tether technology is currently being pursued to dramatically lower the launch mass and cost of interplanetary scientific spacecraft. A spacecraft boosted from LEO to a high-energy orbit by a MXER tether has most of the orbital energy it needs to escape the Earth's gravity well. However, the final targeting of the spacecraft to its eventual trajectory, and some of the unique issues brought on by the tether boost, are the subjects of this paper.

Introduction

MXER tether technology is currently being developed by NASA through the Office of Space Science (OSS) because MXER tether technology has the potential to augment the performance of most interplanetary missions. The MXER tether concept is as follows:

A MXER tether facility consists of a ~100 km high-strength tether with a number of "control stations" distributed near the "ballast" end of the tether and a "catch mechanism" on the "tip" of the tether. The tether facility is in an equatorial Earth orbit of roughly 400 km perigee by 8800 km apogee. The tether rotates fairly rapidly (about once every 6 minutes) such that its tip can match position and velocity (but not acceleration) with a spacecraft in a circular equatorial low Earth orbit (LEO). During this moment of proximity, the tether "catches" the spacecraft, and the spacecraft is accelerated by the tether for next few minutes. Approximately 3 minutes (1/2 rotation) later, the payload is released from the tether into a new, higher-energy orbit. During this momentum-exchange process, the payload gains substantial orbital angular momentum and energy at the expense of the tether facility.

The tether facility now must restore the energy and momentum imparted to the payload. It does this in a novel manner by driving electrical current through the tether, against the induced voltage gradient, in order to provide thrust to reboost the tether facility. The power required to drive the electrical current is provided by solar arrays; the momentum comes from an interaction with the Earth's magnetic field. Simply put, the tether pushes against the Earth using energy collected from the Sun.

Calculating the Conditions for Hyperbolic Injection

The payload, once "thrown" by the tether, is in a highly elliptical Earth orbit. Current MXER tether designs at NASA are focusing on throwing the payload into an equatorial geosynchronous transfer orbit (GTO). The GTO orbit has its perigee at LEO altitudes (400-500 km) and its apogee at GEO altitude (36,000 km). For communications satellites bound for GEO, the final stage of the process would be to coast until the spacecraft reaches apogee of its GTO orbit and then use an onboard chemical propulsion system to circularize their orbits at the GEO altitude.

However, for spacecraft destined for interplanetary trajectories, they must configure their orbits for a hyperbolic Earth escape trajectory. The conditions for this hyperbolic injection are calculated from the interplanetary trajectory previously determined for that mission. The Mars Exploration Rover-A, launched on June 10, 2003, will be used as an example. The first and second stages of its Delta II booster placed it into a low Earth orbit. Then a third stage attached to the
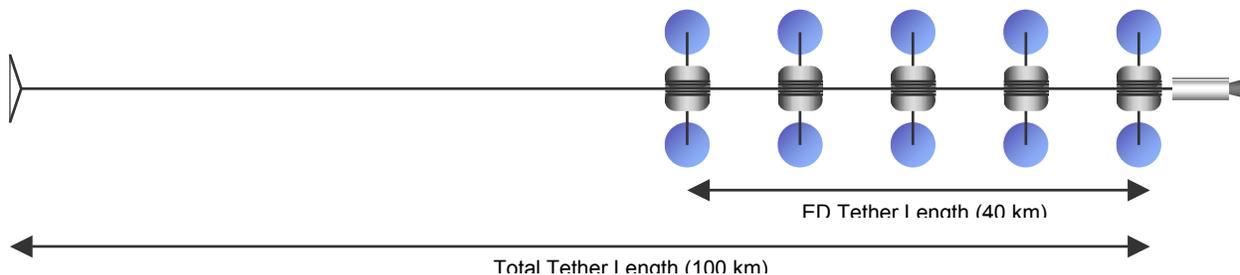
ED Tether Length (40 km)

Total Tether Length (100 km)

spacecraft ignited to send it on a hyperbolic Earth escape trajectory that would take it to Mars. For that particular launch date, JPL mission planners had concluded on an arrival at Mars of January 4, 2004. Based on those two dates (June 10 and January 4), mission planners calculated the positions of Earth and Mars and also the time-of-flight (208 days) between the departure body (Earth) and the arrival body (Mars).

The problem of calculating an orbit given two position vectors and time-of-flight is well known in astrodynamics as Lambert's problem and there are a number of iterative solutions available to quickly find an answer. Given the position vector of Earth on June 10, 2003 and the position vector of Mars on January 4, 2004 and a time-of-flight of 208 days, the orbit that
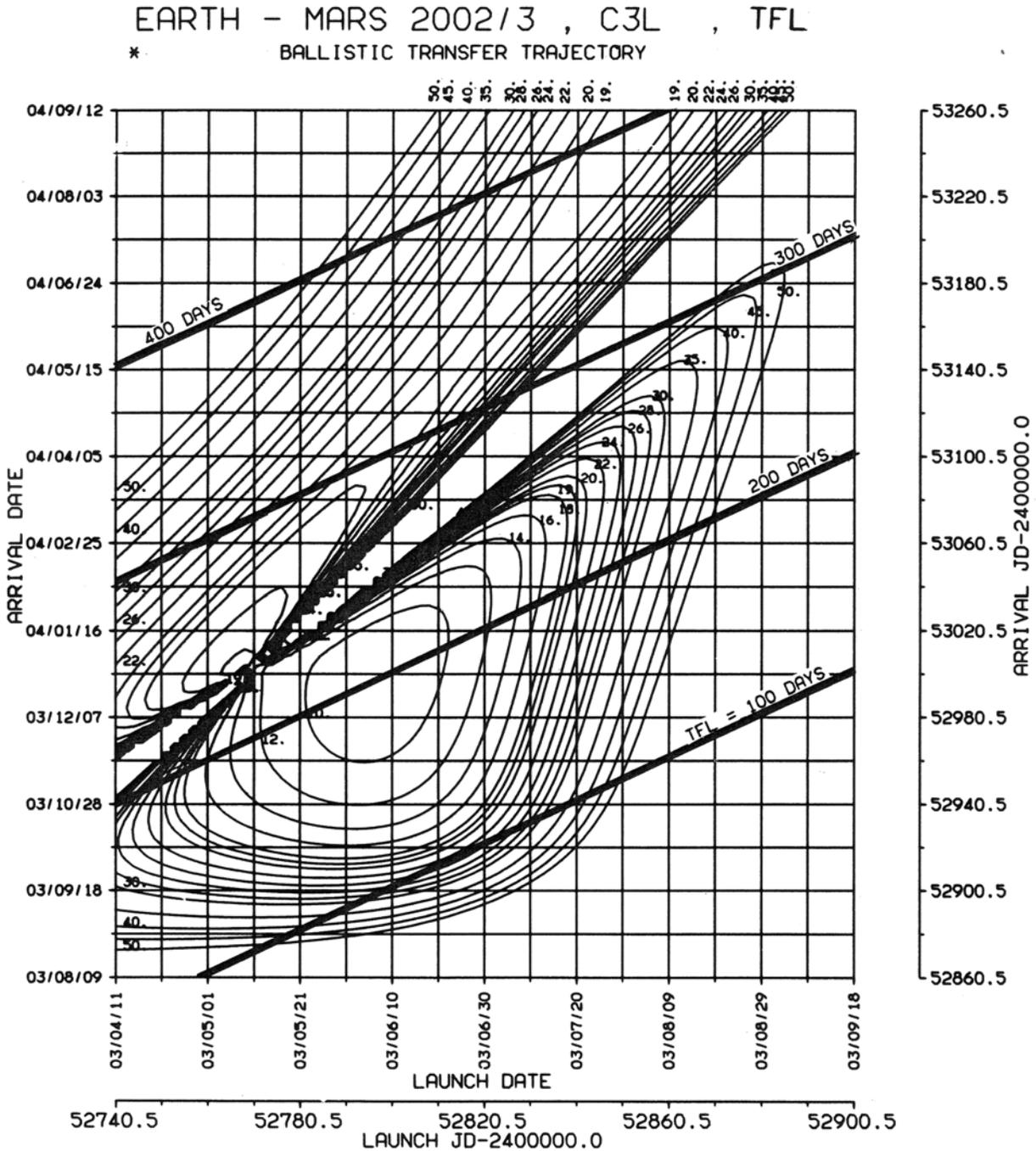

Figure 1: "Porkchop" Plot of Earth-Mars launch opportunities in 2003.

corresponds to those conditions can be calculated, and more importantly, the orbital *velocity* at each of those position vectors can be calculated. If the orbital velocity vector of the Earth is subtracted from the orbital velocity vector of the spacecraft on the departure day, a relative velocity vector is calculated. This relative velocity vector, often called the "V-infinity" vector, is a three-dimensional representation of the departure vector the spacecraft must achieve in order to reach its destination on the desired day.

In summary, the procedure to obtain the V_∞ vectors for hyperbolic departure is:

1. For a given set of departure and arrival dates between two objects, calculate their position and velocity vectors and the time-of-flight. The position vectors will be used in Lambert's problem and the velocity vectors will be used to solve for the V_∞ vectors.
2. Use one of the Lambert problem solution techniques (universal variable, P-iteration, etc.) to calculate velocity vectors at start and end of the trajectory. The initial position and velocity vectors of the trajectory can then be used to calculate the transfer orbit, if desired.
3. Subtract the departure planet's velocity vector from the transfer orbit's initial velocity vector to obtain the relative velocity vector at departure, or V_∞ vector. Subtract the destination planet's velocity vector from the transfer orbit's final velocity vector to calculate the V_∞ vector at arrival.
4. Transform the V_∞ vectors through a matrix representing the orientation of the planet's pole to transform the vector from heliocentric-ecliptic space to planetocentric space. This transformation will not change the magnitude of the vector, only its orientation.
5. Transform the V_∞ vectors from Cartesian to spherical coordinates to obtain the C3 (vector magnitude squared), right ascension angle (longitude), and declination angle (latitude).

Of course, this entire process presupposes that one has already found the optimal departure and arrival dates. *Optimal* is a relative term; although minimizing the energy required to fly the trajectory is a paramount concern, there are additional issues, such as launch window width, landing conditions at the arrival planet, angles between the Earth and Sun during the trajectory, and a host of other concerns that modify the definition of optimum.

In order to understand the trade space quickly, mission planners use contour plots affectionately referred to as "porkchop" plots to visualize the trade space of departure and arrival dates between two planets, as shown in Figure 1.

This example of a "porkchop" plot shows that the departure energies for Earth reach a minimum between about May 20, 2003 and June 22, 2003. A launch on or about these dates leads to a Mars arrival between about November 15, 2003 and January 25, 2004. On June 10, 2003, the third stage of the Delta II rocket injected the Mars Exploration Rover-A spacecraft into a hyperbolic orbit that left the Earth into interplanetary space; the outgoing asymptote of that hyperbola was aligned with the previously calculated V_∞ vector.

Achieving the Calculated Hyperbolic Injection

Once the V_∞ vector is known the problem then becomes one of injecting the spacecraft from some Earth orbit into the desired hyperbolic orbit. There is some degree of flexibility in this because there are a number of hyperbolas whose outgoing asymptotes are identical, as shown in Figure 2. In fact, one can imagine taking a hyperbola of a given periapsis radius and eccentricity and rotating it about the outgoing asymptote to generate a locus of injection points to achieve one of these departure hyperbolas.

If one imagines the center of this locus projected on the surface of the Earth, it would represent the point that is the vectorial inverse of the outgoing asymptote (V_∞). Hence, if one had calculated a trajectory like that of the MER-A, with a DLA (declination of the launch asymptote) of 2.63º, the projection of the center of the locus on the surface of the Earth would be at 2.63º *south* latitude.

The angular radius of the locus (later referred to as the angular extent) is given by the turning angle in the hyperbola. This angular extent of the locus is small for hyperbolic orbits with low eccentricity and grows for orbits with higher eccentricity, as shown by the following table.

C3 value (km^2/s^2)	Orbital Eccentricity	Angular extent of locus (deg)
1	1.017	10.58
2	1.035	14.85
4	1.069	20.72
8	1.138	28.53
12	1.207	34.08
16	1.277	38.43
20	1.346	42.00
40	1.691	53.75
80	2.383	65.18

Turning angle calculated for a perigee radius of 1.08 Earth radii (510.3 km altitude).
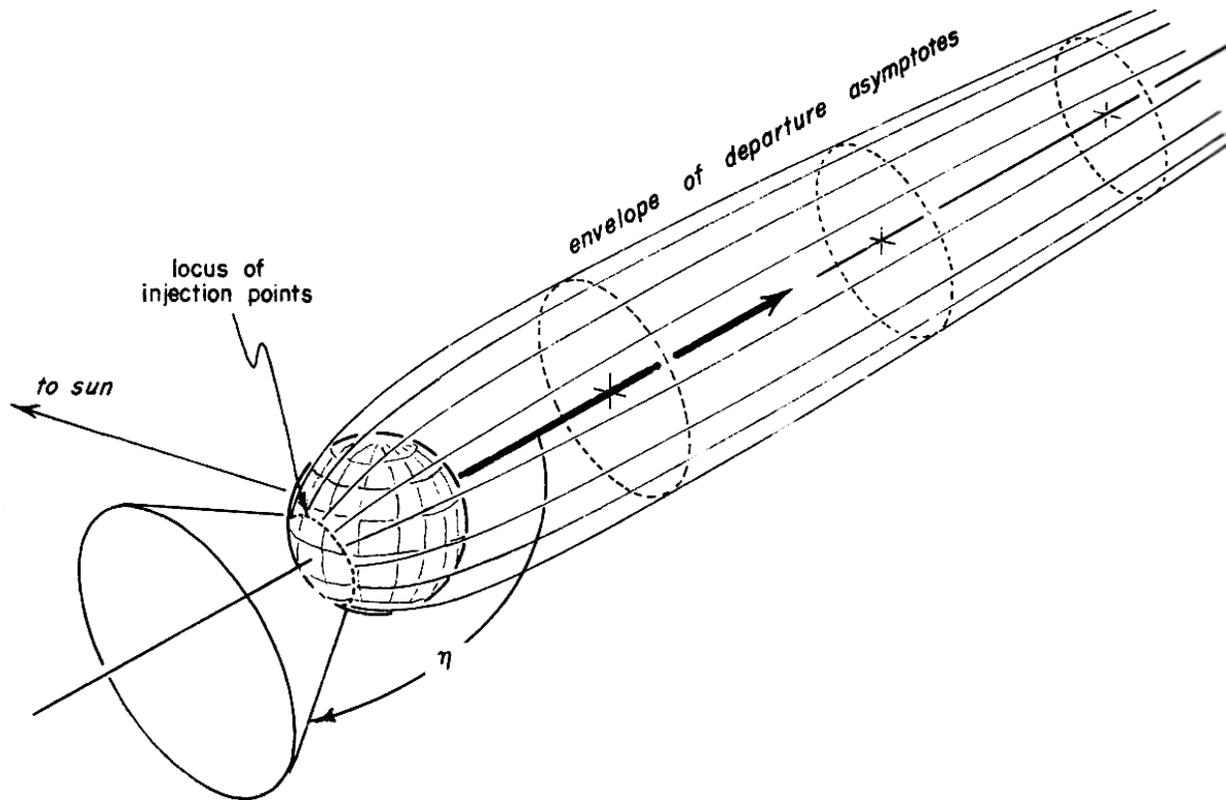
Figure 2: Graphical depiction of a family of hyperbolas that parallel the departure asymptote[1].

However, most interplanetary trajectories require enough C3 to yield a significant locus angular extent. C3 values for direct injection range from 6-12 km^2/s^2 for Venus, 8-20 km^2/s^2 for Mars, and 75-90 km^2/s^2 for Jupiter; additionally, these values are for good launch conditions and low-energy trajectories. Direct injection to planets beyond Jupiter is rarely contemplated because the C3 values are so high that gravity assist trajectories from Venus and/or Jupiter give vastly better performance.

Returning to the original problem, it is assumed that the MXER tether facility will boost the payload up to an equatorial GTO with an arbitrary orientation within that equatorial plane. For sake of initial simplicity, assume that the line of apsides of the GTO (line between periapsis and apoapsis) can be whatever is required.

Recall the locus of hyperbolic injection points is formed by the locations of periapses of the outgoing hyperbola. If the change in velocity (ΔV) between the elliptical GTO and the hyperbolic orbit can be assumed to be instantaneous, then the GTO and the hyperbolic orbit will have identical periapses. For the tether-injected GTO, that periapsis will lie directly over the equator, since the GTO is equatorial. But there is an additional

constraint that will nearly always force the orbit to be non-equatorial, and that is the requirement that the orbit pass directly over the center of the locus.
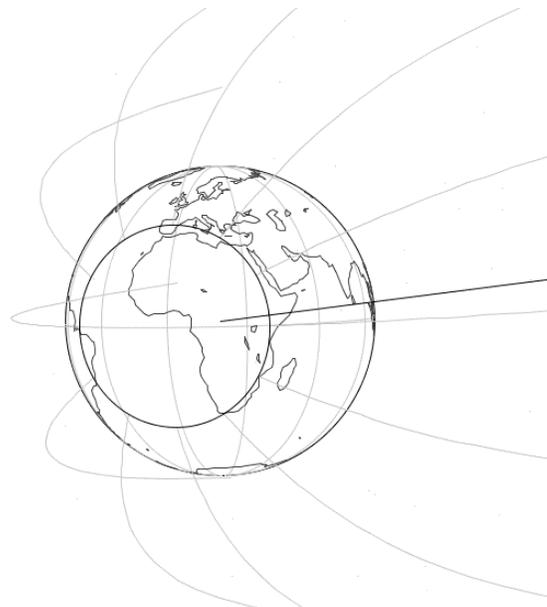

Figure 3: Locus of injection points and departure asymptotes for a 0° declination injection.

It can be deduced from examination that any of the hyperbolas emanating from the locus is coplanar with the outgoing asymptote and the center of the locus itself. Hence, if the injection ΔV is desired to be coplanar with the previous elliptical orbit (and that is *strongly* desirable to minimize ΔV) then the hyperbolic orbit, and the elliptical orbit that preceded it, must pass over the center of the injection locus.

This levies an additional constraint on a payload that is in an initially equatorial orbit. Unless the declination of the launch asymptote is exactly 0º (as shown in Figure 3) it will be necessary to execute a ΔV to change the orbital inclination of the GTO. The inclination necessary is a function of the departure C3 and the DLA.

If inclination changes are conducted at apoapsis of the elliptical orbit (which is strongly desired to minimize ΔV), then the periapsis of the orbit will still remain over the equator. Explained another way, the equatorial elliptical orbit does not have a defined line of nodes (the line formed from the intersection of the orbit and the equatorial plane). When an inclination-change ΔV is executed at apoapsis, the orbit is no longer equatorial, but the line of nodes (which is always in the equatorial plane) and the line of apsides are collinear.
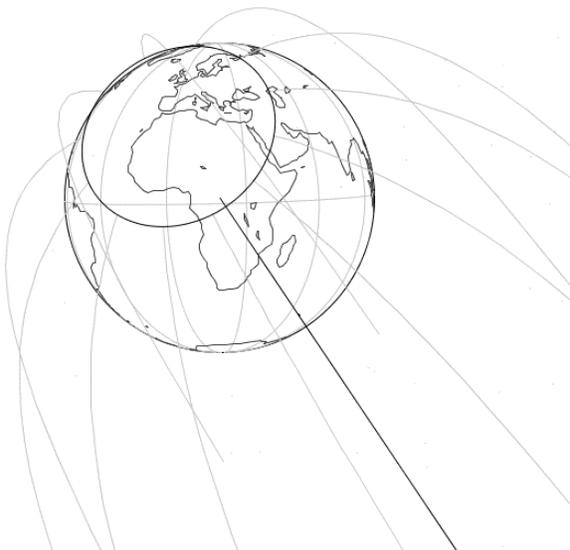
Figure 4: Locus of injection points and departure asymptotes for a –30º declination showing two possible equatorial injection opportunities.

Hence, the heart of the problem is to define a hyperbolic injection opportunity that passes over the equator. If the angular extent (in degrees) of the locus of hyperbolic injection is greater than the declination of the launch asymptote, then the locus will pass over the equatorial plane in at least two locations, as shown in Figure 4, and

two equatorial injections will be possible. If the radius of the locus is equal to the declination of the launch asymptote, then the locus will pass over the equatorial plane in only one location, and the only possible orbit to achieve equatorial injection will be polar. And finally, if the radius of the locus is less than the declination angle, then the locus will never pass over the equator, and equatorial injection into that hyperbolic trajectory will be impossible. This case is shown in Figure 5.

In simpler terms, there will be some trajectories whose hyperbolic departures are such that a tether-boosted spacecraft would be extremely penalized to try to achieve them. Fortunately, most trajectories of interest do not fall in this category.

Figure 5: Locus of injection points and departure asymptotes for a –45º declination showing no feasible equatorial injection opportunities.

Most solar system targets of interest, such as Venus, Mars, and Jupiter, lie roughly in the ecliptic plane, and the Earth is only tilted 23.5º degrees to the ecliptic. If all solar system targets were perfectly coplanar, then injection declination would range from a maximum of 23.5º to a minimum of -23.5º throughout the year.

However, since Venus (3.4º), Mars (1.8º), Jupiter (1.3º) and other targets are not coplanar with the Earth, heliocentric plane change is a factor in interplanetary injection. Sometimes this plane change requirement can reduce the injection declination; sometimes it can make it much higher.

Additionally, most of these injection trajectories have enough C3 requirements to cause their loci of hyperbolic injection points to have a fairly large angular extents. This means that most of these trajectories have

feasible equatorial injection opportunities. For instance, MER-A, launched on June 10, 2003, had a DLA of 2.63º and a C3 of 8.9 km²/s²; with that C3, the angular extent of the locus is 29.5º at an injection altitude of 510 km.

Calculating the Orbital Elements of the Injection Hyperbola

Given a desired C3, right ascension, and declination of the hyperbolic injection asymptote, the following procedure will yield the orbital elements of a hyperbolic orbit that will match that departure asymptote and have an equatorial periapsis.

To begin, the desired periapsis altitude must be chosen. ΔV requirements will be minimized if the periapsis altitude is minimized, but this altitude must be above any atmosphere as well. Once determined, the periapsis velocity can be calculated from the C3 and periapsis radius using the equation:

$$v_p = \sqrt{C3 + \frac{2\mu}{r_p}}$$

This procedure to calculate the orbital elements is then:

1. Eccentricity (e) of the hyperbola is calculated from C3.
2. Semi-major axis (a) of the hyperbola (which will be negative) is calculated from periapsis radius and eccentricity.
3. True anomaly (υ) will be 0º since the injection will be at periapsis.
4. Argument of the periapsis (ω) will be 0º or 180º since the line of apsides lies in the equatorial plane.
5. The orbital inclination (i) is calculated from spherical trigonometry using the angular extent of the departure locus and the declination of the launch asymptote.
6. Right ascension of the ascending node (Ω) is also calculated using spherical trigonometry from the angular extent of the departure locus, the declination of the launch asymptote, and the right ascension of the launch asymptote.

First calculate eccentricity of the hyperbola from the C3 of the injection asymptote, the radius of periapsis, and the gravitational parameter of the attracting body.

$$e = 1 + \frac{C3 \cdot r_p}{\mu}$$

The eccentricity should be greater than one since the orbit is a hyperbola.

Next, calculate semi-major axis from periapsis radius and eccentricity:

$$a = r_p(1 - e)$$

The semi-major axis will have a negative distance since the orbit is hyperbolic.

Spherical trigonometry allows us to calculate the orbital inclination for the special case of an equatorial periapsis. The angular extent of the injection locus is given by

$$\cos\eta = \frac{1}{e}$$

If the declination (ϕ) of the injection asymptote is less than the angular extent of the injection locus, then a solution is feasible, as shown in Figure 4. In this case, the orbital inclination can be calculated from the following expression.

$$\sin i = \frac{\sin\phi}{\sin\eta}$$

The right ascension of the ascending node is calculated in a similar way from spherical trigonometry.

$$\Omega = \theta + \pi + \arccos\left(\frac{\cos\eta}{|\cos\phi|}\right)$$

In this equation, θ represents the right ascension of the hyperbolic asymptote and π is present to "flip" the ascending node to the opposite side of the planet from the hyperbolic asymptote. Recall that there are two equatorial solutions to departure—one will have a prograde inclination and one will have a retrograde inclination. Obviously, only a prograde inclination would be acceptable for a real injection case. Hence, it must determined whether the hyperbolic injection will take place at the ascending node or the descending node of the orbit.

This is quite simply done. If the declination of the hyperbolic asymptote is negative, then the injection locus will lie in the Northern Hemisphere, and a prograde injection will take place on the descending node. Conversely, if the declination is positive, then the locus will lie in the Southern Hemisphere, and a

prograde injection will take place on the ascending node.

Since the orbital element of interest is the right ascension of the *ascending node*, it is necessary to "flip" Ω 180º in the case of negative declination. Also, the argument of periapsis, ω, will be 0º for the positive declination, ascending node case and 180º for the negative declination, descending node case.

With these six orbital elements, it is also straightforward to quickly calculate the radius and velocity vectors at periapsis for equatorial injection:

$$r_x = sr_p \cos \Omega \qquad\qquad v_x = -sv_p \sin \Omega \cos i$$

$$r_y = sr_p \sin \Omega \qquad\qquad v_y = sv_p \cos \Omega \cos i$$

$$r_z = 0.0 \qquad\qquad\qquad v_z = sv_p \sin i$$

In these equations, s is a parameter that is 1 when ω is 0º and is −1 when ω is 180º. It represents the simplification that is possible when ω is either 0º or 180º. All terms involving sin ω drop out and terms involving cos ω are either multiplied by 1 or −1.

Summary

The problem of hyperbolic injection from a tether-assisted equatorial orbit essentially boils down to a problem in defining a hyperbolic injection that has an equatorial periapsis. It has been shown that equatorial solutions are possible when the angular extent of the hyperbolic injection locus is greater than the declination of the launch asymptote. There are two solutions, prograde and retrograde, of which the prograde solution is obviously the most desirable. The orbital elements of this injection hyperbola are obtainable from the orientation of the departure asymptote via spherical trigonometry. These orbital elements can then be used to construct the radius and velocity vectors of this departure hyperbola.

References

1. Bate, R., Mueller, D., White, J. Fundamentals of Astrodynamics, Dover Publications, 1971.

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